Identification of Hydrodynamic Coefficients using Support Vector Regression

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***Abstract*—Numerical models are widely used in the design, analysis and research of ship motions as they allow the study of dynamics of a vessel without the need for expensive physical models. This work aims to accurately build an Abkowitz ship manoeuvring model by using system identification to derive the hydrodynamic coefficients that define the dynamics of a vessel. Support Vector Regression is employed to predict the hydrodynamic coefficients of a Mariner class vessel using turning circle and zigzag maneuvering trial data. The identified coefficients are validated by comparing manoeuvring simulations generated from the regressed and experimental PMM derivatives.**

***Index Terms*—Ship manoeuvring, System Identification, Support Vector Regression, Abkowitz model**

I. Introduction

Mathematical models are widely used in the study of ship manoeuvring mainly because it provides a no cost framework that can be used to run various experiments to predict ship behaviour in wide varieties of sea states. It provides a starting point for the design, engineering and production of new ships and vessels. International Maritime Organisation standards dictate the prediction of ship manoeuvrability during the initial design stages. With the development of computer based techniques, simulation based methods have become the preferred approach. Popular numerical ship simulation models include the Abkowitz model [1] and the MMG model [2], and in this work we look to identify the parameters of the Abkowitz model using simulated data. The coefficients of the nonlinear forces and moments in a general numerical maneuvering model are known as hydrodynamic coefficients. These coefficients encapsulate the dynamics of the vessel and they are used to approximate the interactions between the hull and the fluid surrounding it. The higher order coefficients are crucial to describe accurate dynamics in a computer simulation setting, and precise estimation of these is vital.

In order to capture the complex dynamics during high speed manoeuvres, the developed mathematical model must have sufficient nonlinearity to capture the relevant dynamics of the vessel. The hydrodynamic coefficients must be estimated accurately to ensure that the numerical model exhibits dynamics similar to that observed by the real vessel. Traditionally, the coefficients are estimated by physical experiments using special facilities like the towing tank, rotating arm facility or a planar motion mechanism (PMM) facility. However, such facilities are expensive and not always readily available. Empirical formulations have also been developed for multiple series of ships but they are specific to certain types of vessels and are unreliable when accurate representation of the dynamics is needed. In such scenarios, the system identification of the hydrodynamic coefficients from free running model tests are a lucrative option. Since a free running model will experience similar dynamics as the full scale vessel (but for the scale effects), it consists of the necessary information to extract the significant hydrodynamic coefficients.

Support Vector Machine (SVM) is a supervised learning algorithm mainly used for classification and regression developed by Vapnik [3] based on statistical learning theory. In addition to linear regression SVMs can also perform non-linear regression by using the kernel trick, which consists of mapping non linear data into a high dimensional feature space using a kernel function and a linear regression or classification is now possible in this higher dimensional space. SVM when used for regression is also referred to as Support Vector Regression (SVR). SVR provides robust solutions to system identification problems that consist of a large number of features. The method provides good generalisation performance and can quickly find the globally optimum solutions. Zhang et al. [4] used ε-SVR for identification of an Abkowitz model using $20^\circ/20^\circ$ zig-zag tests of a mariner class vessel, where gaussian noise was added into the training samples to damp effects of parameter drift. The study demonstrated good generalisation performance using SVR. Luo et al [5] demonstrated the same for a KVLCC2 ship, where a difference method was used to augment the training data to diminish parameter drift and the SVR hyperparameters were then tuned using particle swarm optimisation. Wang et al. [6] investigated different modelling schemes for ε-SVR identification and concluded that a white-box modelling scheme was ideal for precise parameter estimation. Liu et al. [7] proposed a batch learning scheme to further diminish parameter drift during ε-SVR along with addition of gauissian noise to the training data. Wang et al. [8] studied ε-SVR on both clean data and on data with environmental disturbances. They concluded that system identification in gentle to moderate environmental disturbances are reasonable.

This study looks to identify the hydrodynamic parameters of the Abkowitz model following a white box modelling scheme. Clean maneuvering data is generated without any disturbances and we look to compute back the derivatives by employing SVR. The training maneuvers are simulated back using the regressed derivatives and then comparisons are made. The paper is organized as follows. The Abkowitz maneuvering model is briefly described Section II. The model is then rewritten to formulate the SVR problem. The theory behind ε SVR is discussed in detail in Section III. Finally the training procedure and results are presented in Section IV. A final discussion is made to consolidate the findings in Section V.

II. Abkowitz maneuvering model

A. Equations of Motion

The equations of motion of a 3-degrees of freedom non-dimensional Abkowitz ship model are described in (1). The degrees of freedom are for longitudinal / surge, transverse / sway and yaw motions. A global inertial frame $O_0{-}X_0, Y_0, Z_0$ and a body-fixed frame of reference $O{-}X, Y, Z$ with origin on waterline mid-ship and z-axis pointing vertically downwards are defined. The body coordinate x-axis points toward bow of the ship and y-axis towards starboard. Initially the x-y plane of the two frames coincide.

$$\begin{bmatrix} m' - X'_{\dot{u}} & 0 & 0 \\ 0 & m' - Y'_{\dot{v}} & m'x'_G - Y'_{\dot{r}} \\ 0 & m'x'_G - N'_{\dot{v}} & I'_{zz} - N'_{\dot{r}} \end{bmatrix} \begin{bmatrix} \dot{u}' \\ \dot{v}' \\ \dot{r}' \end{bmatrix} = \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \tag{1}$$

The super-scripted variables in all equations are considered dimensionless. The prime non-dimensionalised quantities are defined as:

$$\begin{aligned}
m' &= \frac{m}{0.5\ \rho L^3} & x'_G &= \frac{x_G}{L} & \delta' &= \delta \\
u' &= \frac{u}{U} & v' &= \frac{v}{U} & r' &= \frac{r'L}{U} \\
\dot{u}' &= \frac{\dot{u}L}{U^2} & \dot{v}' &= \frac{\dot{v}L}{U^2} & \dot{r}' &= \frac{\dot{r}L}{U^2} \\
X'_{\dot{u}} &= \frac{X_{\dot{u}}}{\frac{1}{2}\rho L^3} & Y'_{\dot{v}} &= \frac{Y_{\dot{v}}}{\frac{1}{2}\rho L^3} & Y'_{\dot{r}} &= \frac{Y_{\dot{r}}}{\frac{1}{2}\rho L^4} \\
N'_{\dot{v}} &= \frac{N_{\dot{v}}}{\frac{1}{2}\rho L^4} & N'_{\dot{r}} &= \frac{N_{\dot{r}}}{\frac{1}{2}\rho L^5} & I'_{zz} &= \frac{I_{zz}}{\frac{1}{2}\rho L^5}
\end{aligned}$$

Here ρ is the density of the fluid, L is the length of the ship, U is the net forward speed of the ship, m is the mass, I_{zz} is the moment of inertia of the ship about the z-axis (Yaw-axis) and x_G is the location of the longitudinal center of gravity in body coordinate frame. u, v and r are the longitudinal velocity, the transverse velocity and the yaw rate of the ship respectively. δ denotes the rudder angle. $\dot{u}$, $\dot{v}$ and $\dot{r}$ are the corresponding first order velocity derivatives in the body coordinate system. $X'_{\dot{u}}$, $Y'_{\dot{v}}$, $Y'_{\dot{r}}$, $N'_{\dot{v}}$ and $N'_{\dot{r}}$ are the non dimensional hydrodynamic added mass derivatives.

X', Y' and Z' in (1) are the net transverse force, longitudinal force and Yaw moment acting on the ship and are expressed in the body coordinate system. Since the forces and moments depend on the interaction between the vessel and fluid surrounding it, we can express them as a Taylor series in u, v, r and δ as shown below:

$$\begin{aligned} X' = {} & X'_u u' + X'_{uu} u'^2 + X'_{uuu} u'^3 + X'_{vv} v'^2 + \\ & X'_{rr} r'^2 + X'_{vr} v'r' + X'_{\delta\delta}\delta'^2 + \\ & X'_{\delta\delta u}\delta'^2 u' + X'_{v\delta} v'\delta' + X'_{v\delta u} v'\delta' u' \end{aligned} \tag{2}$$

$$\begin{aligned} Y' = {} & Y'_v v' + Y'_r r' + Y'_{vvv} v'^3 + Y'_{vvr} v'^2 r + \\ & Y'_{vu} v'u' + Y'_{ru} r'u' + Y'_\delta \delta' + Y'_{\delta\delta\delta}\delta'^3 + \\ & Y'_{\delta u}\delta' u' + Y'_{\delta uu}\delta' u'^2 + Y'_{v\delta\delta} v'\delta'^2 + \\ & Y'_{vv\delta} v'^2\delta' + Y'_0 + Y'_{0u} u' + Y'_{0uu} u'^2 \end{aligned} \tag{3}$$

$$\begin{aligned} N' = {} & N'_v v' + N'_r r' + N'_{vvv} v'^3 + N'_{vvr} v'^2 r + \\ & N'_{vu} v'u' + N'_{ru} r'u' + N'_\delta \delta' + N'_{\delta\delta\delta}\delta'^3 + \\ & N'_{\delta u}\delta' u' + N'_{\delta uu}\delta' u'^2 + N'_{v\delta\delta} v'\delta'^2 + \\ & N'_{vv\delta} v'^2\delta' + N'_0 + N'_{0u} u' + N'_{0uu} u'^2 \end{aligned} \tag{4}$$

It should be noted that the Taylor expansion is taken about $u = u_0$, $v = 0$ and $r = 0$, where u_0 is the forward speed of the ship. For the steady speed u_0 consider a small change in speed Δu, then the net longitudinal velocity in body-coordinate frame becomes $u = u_0 + \Delta u$. Similarly the net transverse velocity is $v = \Delta v$, yaw rate $r = \Delta r$ and rudder angle $\delta = \Delta\delta$. The resultant velocity can then be calculated as:

$$U = \sqrt{(u_0 + \Delta u)^2 + v^2} \tag{5}$$

The equations of motion of the Abkowitz model in (1) are written with both the variables and the hydrodynamic coefficients in the non dimensional form for regression using free model trial data. The equations of motion are expanded out as shown:

$$\begin{aligned} (m' - X'_{\dot{u}})\dot{u}' = {} & X'_u u' + X'_{uu} u'^2 + X'_{uuu} u'^3 + X'_{vv} v'^2 \\ & + \ X'_{rr} r'^2 + X'_{vr} v'r' + X'_{\delta\delta}\delta'^2 \\ & + \ X'_{\delta\delta u}\delta'^2 u' + X'_{v\delta} v'\delta' + X'_{v\delta u} v'\delta' u' \end{aligned} \tag{6}$$

$$\begin{aligned} (m' - Y'_{\dot{v}})\dot{v}' + (m'x'_G - Y'_{\dot{r}})\dot{r}' = {} & Y'_v v' + Y'_r r' + Y'_{vvv} v'^3 \\ & + \ Y'_{vvr} v'^2 r' + Y'_{vu} v'u' + Y'_{ru} r'u' \\ & + \ Y'_\delta \delta' + Y'_{\delta\delta\delta}\delta'^3 + Y'_{\delta u}\delta' u' + Y'_{\delta uu}\delta' u'^2 \\ & + \ Y'_{v\delta\delta} v'\delta'^2 + Y'_{vv\delta} v'^2\delta' + Y'_0 + Y'_{0u} u' + Y'_{0uu} u'^2 \end{aligned} \tag{7}$$

$$
\begin{aligned}
(m'x'_G - N'_{\dot{v}})\dot{v}' + (I'_{zz} - N'_{\dot{r}})\dot{r}' &= N'_v v' + N'_r r' + N'_{vvv} v'^3 \\
&+ N'_{vvr} v'^2 r' + N'_{vu} v' u' + N'_{ru} r' u' \\
&+ N'_\delta \delta' + N'_{\delta\delta\delta} \delta'^3 + N'_{\delta u} \delta' u' + N'_{\delta uu} \delta' u'^2 \\
&+ N'_{v\delta\delta} v' \delta'^2 + N'_{vv\delta} v'^2 \delta' + N'_0 + N'_{0u} u' \\
&+ N'_{0uu} u'^2
\end{aligned} \tag{8}
$$

B. Model Reconstruction for SVR

For parameter identification the non-linear Abkowitz model can be rewritten into a linear form with respect to the hydrodynamic coefficients. Equations (6) - (8) are written in the following form:

$$A(k) \cdot X_c = (m' - X'_{\dot{u}})\ \dot{u}'(k) \tag{9}$$

$$B(k) \cdot Y_c = (m' - Y'_{\dot{v}})\ \dot{v}'(k) + (m'x'_G - Y'_{\dot{r}})\ \dot{r}'(k) \tag{10}$$

$$C(k) \cdot N_c = (m'x'_G - N'_{\dot{v}})\ \dot{v}'(k) + (I'_{zz} - N'_{\dot{r}})\ \dot{r}'(k) \tag{11}$$

The accelerations $\dot{u}(k)$, $\dot{v}(k)$ and $\dot{r}(k)$ are discretized and computed at sample time k using an Euler stepping method then non-dimensionalised.

$$
\begin{aligned}
\dot{u}(k) &= \frac{u(k+1) - u(k)}{h} \\
\dot{v}(k) &= \frac{v(k+1) - v(k)}{h} \\
\dot{r}(k) &= \frac{r(k+1) - r(k)}{h}
\end{aligned} \tag{12}
$$

[A, B, C] are the parameter vectors and [X_c, Y_c, N_c] are the coefficient vectors. They are defined as:

$$
\begin{aligned}
A(k) = \Big[& u'(k), u'(k)^2, u'(k)^3, v'(k)^2, r'(k)^2, v'(k)r'(k), \\
& \delta'(k)^2, \delta'(k)^2 u'(k), v'(k)\delta'(k), \\
& v'(k)\delta'(k)u'(k) \Big]_{1\times 12}
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
B(k) = \Big[& v'(k),\ r'(k),\ v'(k)^3, v'(k)^2 r'(k),\ v'(k)u'(k), \\
& r'(k)u'(k),\ \delta'(k),\ \delta'(k)^3, \delta' u'(k),\ \delta'(k)u'(k)^2, \\
& v'(k)\delta'(k)^2,\ v'(k)^2\delta'(k),\ 1,\ u'(k), \\
& u'(k)^2 \Big]_{1\times 15}
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
C(k) = \Big[& v'(k),\ r'(k),\ v'(k)^3, v'(k)^2 r'(k),\ v'(k)u'(k), \\
& r'(k)u'(k),\ \delta'(k),\ \delta'(k)^3, \delta' u'(k),\ \delta'(k)u'(k)^2, \\
& v'(k)\delta'(k)^2,\ v'(k)^2\delta'(k),\ 1,\ u'(k), \\
& u'(k)^2 \Big]_{1\times 15}
\end{aligned} \tag{15}
$$

The coefficient vectors containing the longitudinal, transverse and yaw hydrodynamic derivatives are:

$$
\begin{aligned}
X_c = \Big[& X'_u, X'_{uu}, X'_{uuu}, X'_{vv}, X'_{rr}, X'_{vr}, X'_{\delta\delta}, X'_{\delta\delta u}, X'_{v\delta}, \\
& X'_{v\delta u} \Big]^T_{12\times 1}
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
Y_c = \Big[& Y'_v, Y'_r, Y'_{vvv}, Y'_{vvr}, Y'_{vu}, Y'_{ru}, Y'_\delta, Y'_{\delta\delta\delta}, Y'_{\delta u}, Y'_{\delta uu}, \\
& Y'_{v\delta\delta}, Y'_{vv\delta}, Y'_0, Y'_{0u}, Y'_{0uu} \Big]^T_{15\times 1}
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
N_c = \Big[& N'_v, N'_r, N'_{vvv}, N'_{vvr}, N'_{vu}, N'_{ru}, N'_\delta, N'_{\delta\delta\delta}, N'_{\delta u}, N'_{\delta uu}, \\
& N'_{v\delta\delta}, N'_{vv\delta}, N'_0, N'_{0u}, N'_{0uu} \Big]^T_{15\times 1}
\end{aligned} \tag{18}
$$

The derivatives in (16) - (18) are identified by solving equations (9) - (11) using SVR. The identification steps followed is to first collect maneuvering trial data computed using experimental PMM derivatives of a Mariner vessel. The parameter vectors in 13 - 15 are generated keeping all variables non dimensional, then SVR hyper-parameter are tuned and the trained to achieve satisfactory results. Issues related to parametric drift causes the regressed derivatives to deviate from the original, and is caused due to multi-collinearity between the columns of the parameter vector matrix. Technique's to alleviate parametric drift include parameter excitation using random noise, data augmentation and parameter vector restructuring. Research is still ongoing on the quest to achieve accurate derivative predictions using SVR.

III. ε - SUPPORT VECTOR REGRESSION

Support vector regression is preferred for system identification due to its high generalisation and its ability to find globally optimal unique solutions. The algorithm looks to compute a separating hyper-plane by maximising the margin between the data points and itself while allowing for an error margin of ε. In the case of non-linear data, they are mapped to a higher dimensional space using a kernel function. The data then becomes linearly separable in higher dimensions and a linear regression is performed. The value of ε can then be tweaked to accommodate outliers so that a majority of the data can be separated by the generated hyper-plane. A C value parameter is also used for account for regularisation.

Let the multi-dimensional input and target output data be the set $\chi = \{(x_i, y_i) : x_i \in \mathbb{R}^N, y_i \in \mathbb{R}, N \geq 1\}$. Then the SVR [10] general approximation hyperplane is defined by

$$y(x) = \mathbf{w} \cdot \Phi(x) + b \tag{19}$$

where $\Phi(x)$ is the feature map corresponding to some positive definite symmetric kernel K. K is used to map the input data to some higher dimensional space. The optimisation problem is then defined to find the maximum margin hyperplane that can fit the data in this higher dimensional space. The margin is defined to be proportional to $1/||\mathbf{w}||$ and maximising it is equivalent to minimising $||\mathbf{w}||$. The optimisation objective including the empirical loss is then written as follows:

$$\min_{\mathbf{w},b} \frac{1}{2}||\mathbf{w}||^2 + C \sum_{i=1}^{m} |\ y_i - (\mathbf{w} \cdot \Phi(x) + b)\ |_\epsilon \tag{20}$$

where $|\cdot|_\epsilon$ is the ϵ insensitive hinge loss function, m is total size of the training set. Using slack variables $\xi_i \geq 0$ and $\xi_i' \geq 0$, $i \in [m]$, the dual optimisation problem is equivalently written as:

$$\begin{aligned} &\min_{\mathbf{w},b} \frac{1}{2}||\mathbf{w}||^2 + C \sum_{i=1}^{m} (\xi_i + \xi_i') \\ &s.t\ (\mathbf{w} \cdot \Phi(x) + b) - y_i \leq \epsilon + \xi_i \\ &\quad y_i - (\mathbf{w} \cdot \Phi(x) + b) \leq \epsilon + \xi_i' \\ &\quad \xi_i \geq 0,\ \xi_i' \geq 0,\ i \in [m] \end{aligned} \tag{21}$$

The above convex quadratic objective is rewritten using a Lagrangian and then Karsh-Kuhn-Tucker conditions are applied. The dual optimisation objective is then:

$$\max_{\alpha', \alpha} \left[\sum_{i=1}^{m} y_i(\alpha_i' - \alpha_i) - \sum_{i=1}^{m} \epsilon(\alpha_i' + \alpha_i) - \frac{1}{2} \sum_{i=1}^{m} (\alpha_i' - \alpha_i)(\alpha_j' - \alpha_j) K(x_i, x_j) \right] \tag{22}$$

Let α^* and α be the solutions of the Lagrangian variables returned by the optimiser. The separation plane for the dual is then given by:

$$y(x) = \sum_{i=1}^{m} (\alpha_i^* - \alpha_i) K(x_i, x) + b \tag{23}$$

In the above formulation the C value is the penalty factor that governs the trade off between the margin maximisation and slack and acts as a regularization parameter. ϵ determines the precision of the fit and defines an acceptable error boundary. To carry out system identification a linear kernel is selected, $K(x_i, x_j) = x_i \cdot x_j$. Then the regression plane becomes:

$$y(x) = \sum_{i=1}^{m} (\alpha_i' - \alpha_i) x_i \cdot x + b \tag{24}$$

Comparing equation (24) with (9) - (11), we can correlate the derivative vectors to $\sum_{i=1}^{m} (\alpha_i' - \alpha_i) x_i$, which is easily computed after regression and for an ideal fit $b \sim 0$.

IV. Regression Results

A Mariner class vessel is used in the study, the principal parameters of the vessel is given in Table I. The turning circle, $10°/10°$, $15°/15°$ and $20°/20°$ zig-zag tests are mathematically simulated using hydrodynamic coefficients obtained from PMM tests [9]. Longitudinal, transverse velocities and yaw rate (u, v, r) are recorded along with the rudder angle δ. The training data is sampled at a frequency of 2 Hz for a total simulation time of 1200 secs giving a total of 2400 samples. The SVR hyperparameters ε and C are determined via a grid search over the range $[10^{-10}, 0]$ and $[10^3, 10^8]$ respectively. Based on the tests conducted, $\varepsilon = 0$ and $C = 10^8$ were found to be optimal for good performance in this example. Regression is performed in python using the scikit-learn SVR [11] library.

TABLE I: Mariner Class Vessel Principal Particulars

Parameter	Value
Scale	1
Length between perpendiculars (L_{pp})	160.93 m
Breadth (B)	23.17 m
Design draft (d)	8.23 m
Displacement	18541 m^3
Maximum Rudder Rate	$5°/s$
Design Speed	15 knots
Non Dimensional Mass (m')	798×10^{-5}
Non Dimensional Yaw moment of Inertia (I'_{zz})	39.2×10^{-5}
Non Dimensional Longitudinal Center of Gravity (x_G)	-0.023

The hydrodynamic derivatives $X'_{\dot{u}}$, $Y'_{\dot{v}}$, $Y'_{\dot{r}}$, $N'_{\dot{v}}$ and $N'_{\dot{r}}$ described in the equations of motion need to be known before hand for regression. These can be estimated with a decent accuracy through empirical methods or using panel methods. The values of these derivatives corresponding to added mass are tabulated in Table II for the Mariner class vessel. The parameter vector and the target vectors as described in (9) - (11) are generated for each sampled time instance. All the derivatives are non-dimensional, and training data collected including $[u, v, r, \delta]$ are made non-dimensional for computations. The coefficient vectors are then predicted using SVR.

The results of the regression are given in Tables III, IV and V.

TABLE II: Mariner class Added mass coefficients (1×10^{-5})

Added mass	Value
$X'_{\dot{u}}$	-840
$Y'_{\dot{v}}$	-1546
$Y'_{\dot{r}}$	9
$N'_{\dot{v}}$	23
$N'_{\dot{r}}$	-83

TABLE III: Regressed longitudinal coefficients (1×10^{-5})

X-Coef	PMM	SVR	Error (%)
X'_u	-184	-185	-1.0
X'_{uu}	-110	-114	-10.9
X'_{uuu}	-215	-217	-17.2
X'_{vv}	-899	-1013	-14.9
X'_{rr}	18	12	44.4
X'_{vr}	798	748	7.9
$X'_{\delta\delta}$	-95	-94	1.1
$X'_{\delta\delta u}$	-190	-187	2.6
$X'_{v\delta}$	93	94	-3.2
$X'_{v\delta u}$	93	90	-10.8

From the regression results we observe that a majority of the regressed derivatives show satisfactory agreement. While the longitudinal derivatives show good accuracy, a few derivatives of the transverse and yaw coefficients show considerable deviation which we attribute to the influence of parametric drift. We see that the higher order δ derivatives of sway and yaw seem to be particularly prone to the effect. Parametric drift is a prevalent challenge in any system identification problem. The higher order non-linear parameter vectors show multi-collinearity causing the regression solution to become non-unique. This leads to false erroneous solutions that would be able to mimic the training maneuvers to a good extent. The errors will become more pronounced as we start to simulate sharper maneuvers. Precise estimation of the hydrodynamic derivatives thus becomes very a challenging task.

TABLE IV: Regressed transverse coefficients (1×10^{-5})

Y-Coef	PMM	SVR	Error (%)
Y'_v	-1160	-1259	-8.5
Y'_r	-499	-553	-10.8
Y'_{vvv}	-8087	-8629	-6.8
Y'_{vvr}	-15356	-14564	5.2
Y'_{vu}	-1160	-1338	-15.3
Y'_{ru}	-499	-573	-14.8
Y'_{δ}	278	256	7.9
$Y'_{\delta\delta\delta}$	-90	-71	21.1
$Y'_{\delta u}$	556	552	0.7
$Y'_{\delta uu}$	278	360	-29.5
$Y'_{v\delta\delta}$	-4	174	-4250.0
$Y'_{vv\delta}$	1190	1833	-54.0
Y'_0	-4	-4	0.0
Y'_{0u}	-8	-8	0.0
Y'_{0uu}	-4	-4	0.0

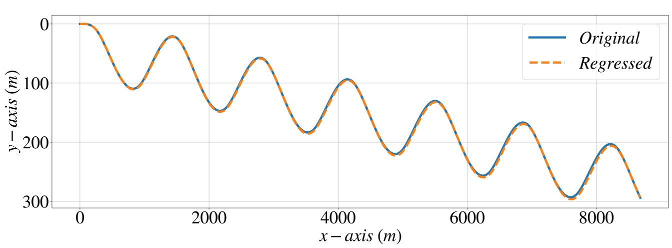
To validate the regression solution we contrast the ship trajectories computed using the real and regressed coefficients. Maneuvers are simulated using the regressed derivatives and compared to the original training maneuvers. The $10°/10°$, $15°/15°$ and $20°/20°$ zig-zag maneuvers are simulated and compared in Fig. 1, 2 and 3. The heading and rudder angles are also plotted against time. We see that both the trajectories and heading angles match closely. The ε-SVR thus is able to show good generalisation capability and has learned to model multiple maneuvers. It is also worth noting that ε-SVR is capable of producing accurate results in a very short period of time given the volume of training data used.

V. Conclusion

In this paper ε-SVR is employed to predict the hydrodynamic coefficients of an Abkowitz maneuvering model. A $10°/10°$, $15°/15°$ and $20°/20°$ zig-zag maneuvers are simulated to train the algorithm. The regressed derivatives are found to match well with the PMM training derivatives. Comparison with the training maneuvers show the generalisation capability of SVR when applied to ship parameter identification. Parameter drift was found to cause error in some of the computed derivatives. In future research we look to produce accurate predictions for all hydrodynamic derivatives by providing solutions to dampen the effects of parametric

TABLE V: Regressed Yaw coefficients (1×10^{-5})

N-Coef	PMM	SVR	Error (%)
N'_v	-264	-234	11.3
N'_r	-166	-151	9.0
N'_{vvv}	-1636	-2205	-34.8
N'_{vvr}	-5483	-5092	7.1
N'_{vu}	-264	-176	33.3
N'_{ru}	-166	-139	16.26
N'_{δ}	-139	-133	4.31
$N'_{\delta\delta\delta}$	45	36	20.0
$N'_{\delta u}$	-278	-286	-2.9
$N'_{\delta uu}$	-139	-180	-26.5
$N'_{v\delta\delta}$	-13	98	-653.8
$N'_{vv\delta}$	-489	-753	-54.0
N'_0	3	3	0.0
N'_{0u}	6	6	0.0
N'_{0uu}	3	4	-33.3



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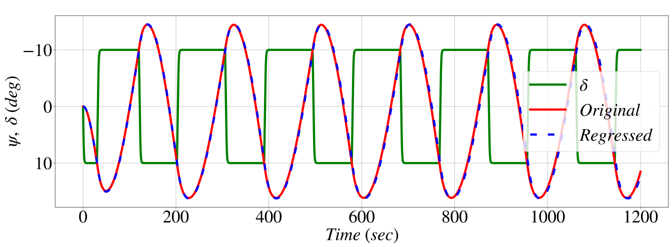
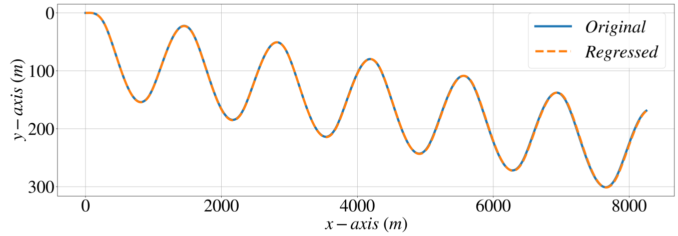


(b)

Fig. 1: Comparison of predicted simulation for $10°/10°$ zig-zag maneuver. (a) Trajectory; (b) Heading (ψ) and Rudder (δ) angle



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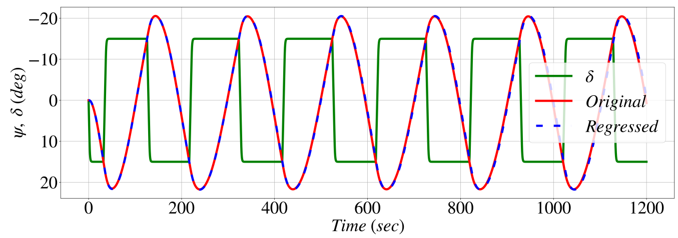
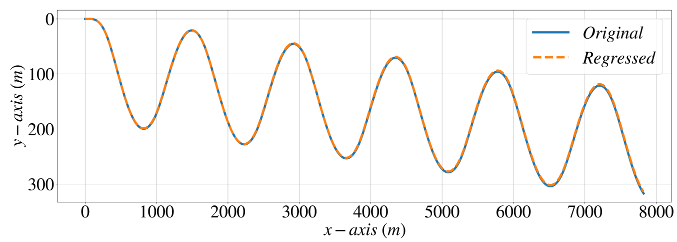
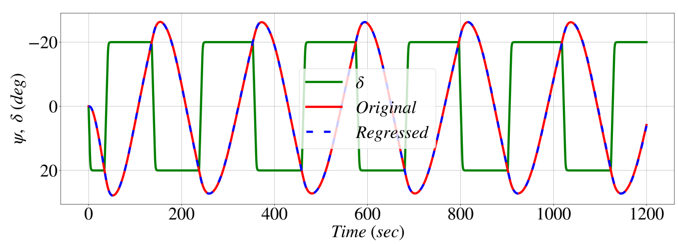


(b)

Fig. 2: Comparison of predicted simulation for $15°/15°$ zig-zag maneuver. (a) Trajectory; (b) Heading (ψ) and Rudder (δ) angle



(a)



(b)

Fig. 3: Comparison of predicted simulation for $20°/20°$ zig-zag maneuver. (a) Trajectory; (b) Heading (ψ) and Rudder (δ) angle

drift. We then look to apply the method to experimental data generated from scaled model trials for prediction and analysis.

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